The Overstrain of Thick-Walled Cylinders Considering the Bauschinger Effect Factor (BEF)

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An independent kinematic hardening material model in which the reverse yielding point is defined by the Bauschinger effect factor (BEF), has been defined for stainless steel SUS 304. The material model and the BEF are obtained experimentally and represented mathematically as continuous functions of effective plastic strain. The material model has been incorporated in a non-linear stress analysis for the prediction of reverse yielding in thick-walled cylinders during the autofrettage process of these vessels. Residual stress distributions of the independent kinematic hardening material model at the onset of reverse yielding are compared with residual stresses of an isotropic hardening model showing the significant effect of the BEF on reverse yielding predictions. Critical pressures of direct and reverse yielding are obtained for the most commonly used cylinders and a range of permissible internal pressures for an efficient auto-frettaged process is recommended.

Key Words : Cylinder, Reverse Yielding, Bauschinger Effect Factor, Autofrettage

1. Introduction

Metals initially overstrained in tension have a significantly lower elastic limit in compression (Bauschinger phenomenon). This can cause reverse yielding to take place at the inside surface of thick-walled cylinders during the autofrettage process of these vessels. Reverse yielding caused by highly compressive residual stresses can affect the performance characteristics and the fatigue strength of these pressure vessels. Chen (1986) presented a closed form solution for the residual stress distribution in a cylinder of high strength steel. He proposed a theoretion constitutive material model in which a perfectly plastic loading condition and a linear hardening unloading function including the Bauschinger effect have been considered. Using this material model and following the procedures in Bland's work (1956), Chen obtained a closed form solution and calculated residual stresses for a few specific values of the BEF. Results of Chen (1986) showed that the magnitudes of compressive residual stresses at the inner surface of the cylinder were substantially decreased when reverse yielding took place in the cylinder. Rees (1987) considered a closed-end cylinders of hardening and non-hardening material model. He assumed that the axial plastic strain is zero, and the radial and tangential plastic strains are equal in magnitude and opposite in sign. With this assumption the Mises effective plastic strain increment was reduced to a function of tangential plastic strain increment and the history dependent problem of plastic stresses were reduced to a numerical integration using uniaxial stress-strain data. Rees (1987) compared the residual stresses of the hardening and non-hardening material model and showed that the hardening model is more realistic and closer to experimental results. Loghman and Wahab (1994) compared

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the residual stress distributions of thick-walled cylinders with and without the effect of gradients considering an isotropic hardening material model. The ANSYS 5 (1992) finite element program provides seven options to define different types of material behaviors two of which exhibit the Bauschinger phenomenon. These are bi-linear kinematic hardening model designated by BKIN and multi-linear kinematic hardening model designated by MKIN. These models are defined based on the conventional assumption that that the total elastic range ($2\sigma_o$) remain constant irrespective of the amount of prior plastic and the material's actual behaviour during reverse yielding. This study incorporates an independent strain-hardening material model in which the reverse yielding point is defined by the BEF. The BEF is a function of the amount of previous overstrain and is obtained experimentally in a manner similar to Milligan et al. (1966).

2. Material Model

A large number of test specimens have been loaded up to a specific strain beyond the elastic limit and then reversely loaded down to zero strain by using a computer-controlled testing machine. As an example, the stress-strain diagram of the material during loading up to 0.75% overstrain and unloading down to zero strain obtained from the experiment is shown in Fig. 1.

Using the material's data file elastic strains at all stress levels are subtracted from the total strains to obtain plastic strains. Using a curve fitting software the strain hardening is mathematically represented by the following constitutive equation :

$$\sigma_e=232.68187+689.01541(\varepsilon_p)^{0.21842186} \quad (1)$$

where σ_e and ε_p are uniaxial stress and plastic strain which have been considered as the effective stress and effective plastic strain respectively. The material reverse yielding point is obtained based on 0.1% offset method and the Bauschinger effect factor is defined as follows :

$$BEF=\frac{\sigma_D}{\sigma_A} \quad (2)$$

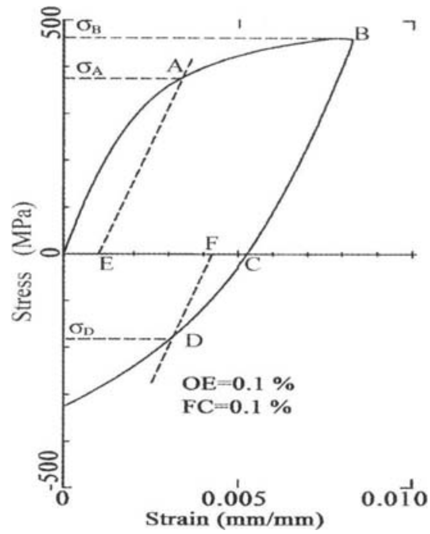

Fig. 1 Experimental loading-unloading stress-strain curve obtained for SUS 304

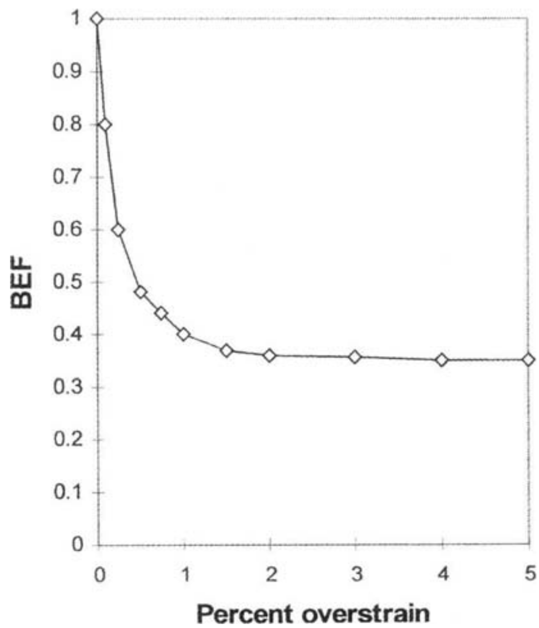

Fig. 2 The experimentally obtained BEF and its approximated function for SUS 304

where σ_A and σ_D are direct and reverse yielding stresses shown in Fig. 1. The BEF is obtained exprimentally and represented mathematically as a continuous function of the percentage amount of plastic overstrain ($\%\varepsilon_p$) as follows :

$$BEF=1.0170029-0.97738304(\%\varepsilon_p)^{0.5}+0.36592732(\%\varepsilon_p)-0.0025343135(\%\varepsilon_p)^3 \quad (3)$$

The experimentally obtained BEF and its mathematical representation are shown in Fig. 2.

3. Theoretical Analysis

Equations of equilibrium, compatibility and stress-strain in cylindrical coordinates are:

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r-\sigma_\theta}{r}=0 \quad \text{(equilbrium)} \quad (4)$$

$$\frac{d\varepsilon_\theta}{dr}+\frac{\varepsilon_\theta-\varepsilon_r}{r}=0 \quad \text{(compatibility)} \quad (5)$$

$$\begin{aligned}\varepsilon_r&=\frac{1}{E}[\sigma_r-v(\sigma_\theta+\sigma_z)]+\varepsilon_r^p\\ \varepsilon_\theta&=\frac{1}{E}[\sigma_\theta-v(\sigma_z+\sigma_r)]+\varepsilon_\theta^p \quad \text{(stress-strain)} \quad (6)\\ \varepsilon_z&=\frac{1}{E}[\sigma_z-v(\sigma_r+\sigma_\theta)]+\varepsilon_z^p\end{aligned}$$

Form the simultaneous solution of the above equations and using the boundary condition of a closed end cylinder the following solution is obtained [4]:

$$\begin{aligned}\sigma_r^p&=U(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)+F(r,a,b,\Delta T,P_o)+G(r,a,b,P_i)\\ \sigma_\theta^p&=V(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)+H(r,a,b,\Delta T,P_o)+R(r,a,b,P_i) \quad (7)\\ \sigma_z^p&=W(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)+M(r,a,b,\Delta T)+N(a,b,P_i)\end{aligned}$$

where σ_r^p, σ_θ^p and σ_z^p are radial, tangential and axial plastic stresses, and ε_r^p, ε_θ^p and ε_z^p are radial, tangential and axial plastic strains, a and b are inner and outer radius, P_i and P_o are inner and outer pressure, and ΔT is the temperature difference. Functions U, V, W, F, H, M, G, R and N are defined in the Appendix. Ignoring functions containing plastic strains U, V and W will result in the following elastic stress distribution as:

$$\begin{aligned}\sigma_r^e&=F(r,a,b,\Delta T,P_o)+G(r,a,b,P_i)\\ \sigma_\theta^e&=H(r,a,b,\Delta T,P_o)+R(r,a,b,P_i) \quad (8)\\ \sigma_z^e&=M(r,a,b,\Delta T)+N(a,b,P_i)\end{aligned}$$

Substituting this elastic stress distribution in the von Mises yield criterion, critical pressure of direct yielding is obtained.

$$(\sigma_r^e-\sigma_\theta^e)^2+(\sigma_\theta^e-\sigma_z^e)^2+(\sigma_z^e-\sigma_r^e)^2=2\sigma_o^2 \quad (9)$$

For pressures beyond the critical pressure, plastic yielding will take place at the inner surface of the cylinder. If a cylinder that has developed plastic strains as a result of an internal pressure P_i is unloaded, then residual stresses will be locked in the cylinder wall. It is obvious that if the cylinder is loaded again with the same internal pressure it will return back to its configuration right before unloading because this loading-unloading is within the yield surface and reversible. On the other hand, if residual. stresses are added to an elastic stress system due to P_i, it will result in the plastic stress distribution before unloading. This can be mathematically represented by the following equations:

$$\begin{aligned}\sigma_r^r+\sigma_r^e&=\sigma_r^p\\ \sigma_\theta^r+\sigma_\theta^e&=\sigma_\theta^p \quad (10)\\ \sigma_z^r+\sigma_z^e&=\sigma_z^p\end{aligned}$$

where σ_r^r, σ_θ^r and σ_z^r are radial, tangential and axial residual stresses, respectively. Reverse yielding predictions have been made by using Von Mises yield criterion for the residual stresses in conjunction with the Bauschinger effect factor as follows:

$$(\sigma_r^r-\sigma_\theta^r)^2+(\sigma_\theta^r-\sigma_z^r)^2+(\sigma_z^r-\sigma_r^r)^2=2(BEF)^2\sigma_o^2 \quad (11)$$

The BEF in this equation is a function of the amount of previous overstrain and therefore is variable through the plastic zone of the cylinder. The reverse yielding prediction is therefore a history dependent problem and the history of loading and consequent plastic stresses and total plastic strains are necessary for residual stresses and reverse yielding prediction. A computer program based on the incremental theory of plasticity has been developed in which the material properties including the BEF are defined by Equations (1) and (3).

4. Numerical Procedure

The used numerical procedure is as follows:

(1) The critical pressure (P_{crit}) of the cylinder is calculated using Von Mises yield criterion Equation (11).

(2) Suppose the final pressure is P_f. The load-

ing path is divided into N steps, each pressure increment is $\Delta p=(P_f-P_{crit})/N$. The internal pressure at ith loading step is then

$$P=P_{crit}+i\Delta p$$

(3) Initial values are assumed for radial and tangential plastic strain increments $\Delta\varepsilon^p_{r,ij}$ and $\Delta\varepsilon^p_{\theta,ij}$ and are added to their accumulated values obtained from the previous loading steps at all division points in the plastic zone along radius. In the initial loading step the accumulated plastic strains are zeroes. The radial, tangential and axial plastic strains are written as:

$$\varepsilon^p_{r,ij}=\sum_{k=1}^{i-1}\Delta\varepsilon^p_{r,kj}+\Delta\varepsilon^p_{r,ij}$$

$$\varepsilon^p_{\theta,ij}=\sum_{k=1}^{i-1}\Delta\varepsilon^p_{\theta,kj}+\Delta\varepsilon^p_{\theta,ij}$$

$$\varepsilon^p_{z,ij}=-(\varepsilon^p_{r,ij}+\varepsilon^p_{\theta,ij}) \qquad \text{(Incompressibility)}$$

where subscripts i and j refer to the loading step and the layer along the radius respectively.

(4) The effective plastic strain increment is then calculated from the associated flow rule of Von Mises yield criterion as follows:

$$\Delta\varepsilon_{p,ij}=\frac{\sqrt{2}}{3}[(\Delta\varepsilon^p_{r,ij}-\Delta\varepsilon^p_{\theta,ij})^2+(\Delta\varepsilon^p_{\theta,ij}-\Delta\varepsilon^p_{z,ij})^2+(\Delta\varepsilon^p_{z,ij}-\Delta\varepsilon^p_{r,ij})^2]^{\frac{1}{2}}$$

(5) Effective plastic strain is then obtained by summation of effective plastic strain increments as:

$$\varepsilon_{p,ij}=\sum_{k=1}^{i}\Delta\varepsilon_{p,ij}$$

(6) From the material constitutive equation the effective stress is calculated at each loading step for all layers along the radius in the plastic zone as:

$$\sigma_{e,ij}=232.68187+689.01541(\varepsilon_{p,ij})^{0.21842186}$$

(7) The radius of elastic-plastic boundary at ith loading step is found by setting the boundary condition at this radius. At the elastic-plastic interface Von Mises yield condition must be satisfied.

(8) Since the elastic-plastic interface is obtained, the integrals of total plastic strains in functions U, V and W shown in the Appendix are calculated using the trapezoid method. Therefore, plastic stresses are calculated from (Eq. 7).

(9) Having the stresses from step 8, the effective plastic strain from step 4, and effective stress from step 6, a new and better approximation is obtained for the latest increment of the plastic strains employing Prandtl-Reuss equations.

$$\Delta\varepsilon^{p(new)}_{r,ij}=\frac{\Delta\varepsilon_{p,ij}}{\sigma_{e,ij}}(2\sigma_{r,ij}-\sigma_{\theta,ij}-\sigma_{z,ij})$$

$$\Delta\varepsilon^{p(new)}_{\theta,ij}=\frac{\Delta\varepsilon_{p,ij}}{\sigma_{e,ij}}(2\sigma_{\theta,ij}-\sigma_{r,ij}-\sigma_{z,ij})$$

$$\Delta\varepsilon^{p(new)}_{z,ij}=-(\Delta\varepsilon^{p(new)}_{r,ij}+\Delta\varepsilon^{p(new)}_{\theta,ij})$$

(10) These new values of plastic strain increments are compared with their assumed values. If convergence has occurred, the procedure continues from step 11. Otherwise, these new obtained values of plastic strain increments are assumed to be the new estimates of plastic strain increments and the above method is repeated from step 3 until convergence take occurs.

(11) If convergence has occurred for ith loading step, then the cylinder is unloaded at this step and residual stresses are calculated as follows:

$$\sigma^r_{r,ij}=\sigma^p_{r,ij}-\sigma^e_{r,ij}$$

$$\sigma^r_{\theta,ij}=\sigma^p_{\theta,ij}-\sigma^e_{\theta,ij}$$

$$\sigma^r_{z,ij}=\sigma^p_{z,ij}-\sigma^e_{z,ij}$$

(12) Reverse yielding is predicted by the Von Mises yield condition including the Bauschinger effect factor as follows:

$$(\sigma^r_{r,ij}-\sigma^r_{\theta,ij})^2+(\sigma^r_{\theta,ij}-\sigma^r_{z,ij})^2+(\sigma^r_{z,ij}-\sigma^r_{r,ij})^2=2(BEF_{ij})^2\sigma_0^2$$

depends on the amount of BEF_{ij} In which the Bauschinger effect factor effective plastic strain as:

$$BEF_{ij}=1.0170029-0.9778304(\%\varepsilon_{p,ij})^{0.5}+0.36592732(\%\varepsilon_{p,ij})-0.0025343135(\%\varepsilon_{p,ij})^3$$

(13) If reverse yielding condition is satisfied, then the internal pressure at this loading step is recorded as the critical pressure for reverse yielding. Otherwise, the loading step is advanced and the numerical procedure is repeated from step 2.

5. Results and Conclusions

An improved material constitutive model based on the actual material behavior including the BEF has been considered in this non-linear

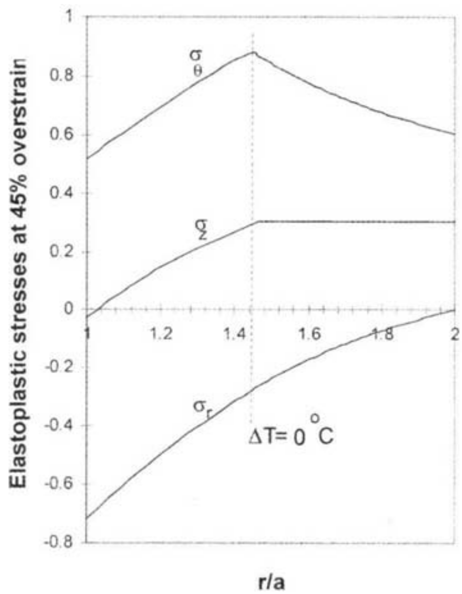

Fig. 3 Elastic-plastic stress distributions of 45% overstrain

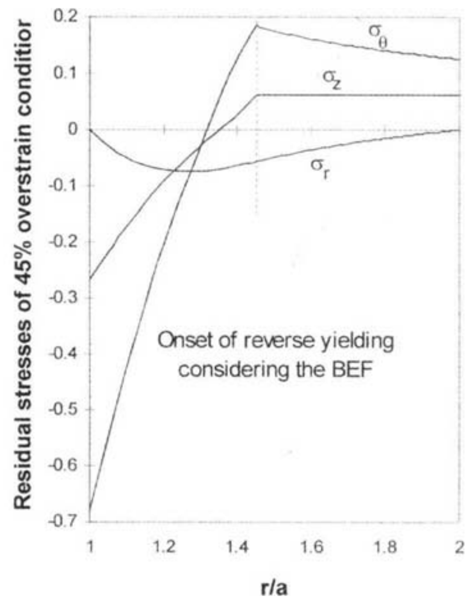

Fig. 5 Residual stress distribution at the onset of reverse yielding considering the BEF

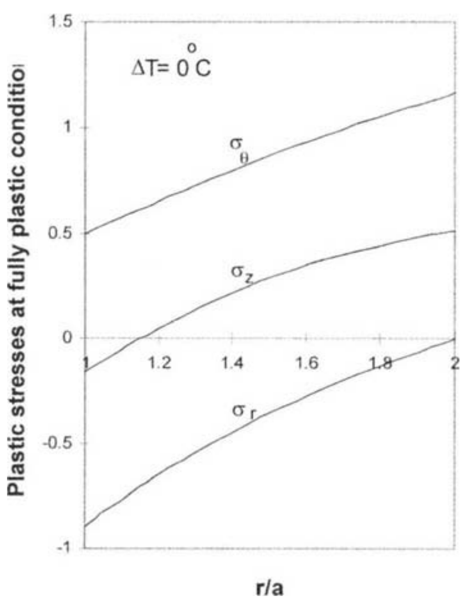

Fig. 4 Plastic stress distribution of fully plastic condition

analysis. The *BEF* is obtained experimentally and represented mathematically as a continuous function of the percentage amount of effective plastic strain. The history of loading, plastic stresses and total plastic strains as well as their consequent residual stresses are recorded during loading-unloading of a thick-walled cylinder. Record of stresses at 45% overstrained and fully plastic conditions are shown in Figs. 3 and 4. Effective plastic strain is variable in the plastic region of the cylinder and therefore the *BEF* is also variable at all division points along radius as plastic deformation progresses. Other investigators have not considered through thickness variation of the *BEF*. The subsequent residual stress distribution of the 45% overstrained condition is shown in Fig. 5. This distribution satisfies the Von Mises yield criterion at the inside surface of the cylinder considering the *BEF*. To show the significant effect of the *BEF* on reverse yielding, residual stresses of the fully plastic condition ignoring the *BEF* are also shown in Fig. 6. It has been concluded that the residual stresses subsequent to 45% overstrain condition are at the onset of reverse yielding when the *BEF* taken into account, but residual stresses resulted from unloading of the same cylinder at the fully plastic condition are not yet at the onset of reverse yielding when the *BEF* is ignored. This is very important for the autofrettage process of thick-walled cylinder. For an efficient autofrettage pro-

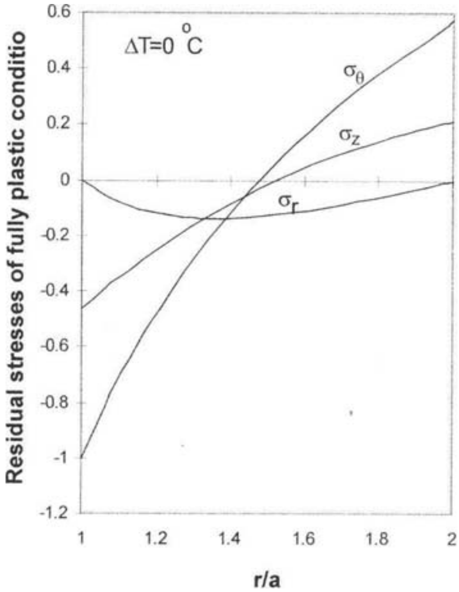

Fig. 6 Residual stress distribution of fully plastic condition ignoring the BEF

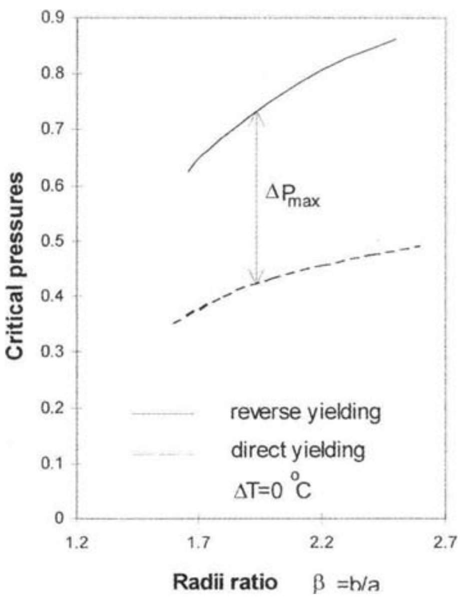

Fig. 7 Critical pressures of direct and reverse yielding

cess a range of permissible pressures (ΔP_{max}) is obtained for a wide range of radii ratios and plotted in Fig. 7 which can be used for practical applications.

Appendix

Functions U, V, W, F, H, M, G, R and N are defined as follows:

$$F(r,a,b,\Delta T,P_o)$$
$$=\frac{E\alpha(\Delta T)}{2(1-v)\,(b^2-a^2)\ln\frac{b}{a}}\left[\frac{a^2b^2}{r^2}\ln\frac{b}{a}+b^2\ln\frac{r}{b}-a^2\ln\frac{r}{a}\right]$$
$$-\frac{P_ob^2}{(b^2-a^2)}\left(1-\frac{a^2}{r^2}\right)$$

$$G(r,a,b,P_i)=\frac{P_ia^2}{(b^2-a^2)}\left(1-\frac{b^2}{r^2}\right)$$

$$H(r,a,b,\Delta T,P_o)$$
$$=\frac{E\alpha(\Delta T)}{2(1-v)\,(b^2-a^2)\ln\frac{b}{a}}\left[\frac{a^2b^2}{r^2}\ln\frac{b}{a}+b^2\ln\frac{r}{b}-a^2\ln\frac{r}{a}+b^2-a^2\right]$$
$$-\frac{P_ob^2}{(b^2-a^2)}\left(1-\frac{a^2}{r^2}\right)$$

$$R(r,a,b,P_i)=\frac{P_ia^2}{(b^2-a^2)}\left(1+\frac{b^2}{r^2}\right)$$

$$M(r,a,b,\Delta T,P_i)$$
$$=\frac{E\alpha(\Delta T)}{2(1-v)\,(b^2-a^2)\ln\frac{b}{a}}\left[2b^2\ln\frac{r}{b}-2\ln\frac{r}{a}+b^2-a^2\right]$$

$$N(a,b,P_i)=\frac{P_ia^2}{(b^2-a^2)}$$

$$U(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)$$
$$=\frac{E}{2(1-v^2)\,(b^2-a^2)}\left[(1-2v)\left(\int_a^b r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr\right)+b^2\left(\int_a^b\frac{\varepsilon_\theta^p-\varepsilon_r^p}{r}\,dr\right)\right]$$
$$\left(1-\frac{a^2}{r^2}\right)$$
$$-\frac{E}{2(1-v^2)\,r^2}\left[(1-2v)\left(\int_a^r r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr\right)+r^2\left(\int_a^r\frac{\varepsilon_\theta^p-\varepsilon_r^{pc}}{r}dr\right)\right]$$

$$V(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)$$
$$=\frac{E}{2(1-v^2)\,(b^2-a^2)}\left[(1-2v)\left(\int_a^b r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr\right)+b^2\left(\int_a^b\frac{\varepsilon_\theta^p-\varepsilon_r^p}{r}\,dr\right)\right]\left(1+\frac{a^2}{r^2}\right)$$
$$-\frac{E}{2(1-v^2)\,r^2}\left[(1-2v)\left(\int_a^r r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr\right)+r^2\left(\int_a^r\frac{\varepsilon_\theta^p-\varepsilon_r^{pc}}{r}dr\right)\right.$$
$$\left.-2r^2((1-v)\,\varepsilon_\theta^p-v\varepsilon_r^p)\right]$$

$$W(r,a,b,\varepsilon_r^p,\varepsilon_\theta^p)$$
$$=\frac{vE}{2(1-v^2)(b^2-a^2)}\left[(1-2v)\left(\int_a^b r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr\right)+b^2\left(\int_a^b \frac{\varepsilon_\theta^p-\varepsilon_r^p}{r}\,dr\right)\right]$$
$$-\frac{vE}{(1-v^2)}\left[(1-v)\varepsilon_\theta^p-v\varepsilon_r^p+\left(\int_a^r \frac{\varepsilon_\theta^p-\varepsilon_r^p}{r}\,dr\right)\right]$$
$$-\frac{E}{b^2-a^2}\left[2\int_a^b r(\varepsilon_\theta^p+\varepsilon_r^p)\,dr-(b^2-a^2)(\varepsilon_\theta^p+\varepsilon_r^p)\right]$$

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